Supplementary material

# A. *Approximation coefficient values*

Table I

Approximation coefficient values within the 12th polyhedron.

|  |  |  |  |
| --- | --- | --- | --- |
| *h* |  |  |  |
| 1 | 1 | 0.2679 | -1 |
| 2 | 1 | 1 | -1.366 |
| 3 | 0.2679 | 1 | -1 |
| 4 | -0.2679 | 1 | -1 |
| 5 | -1 | 1 | -1.366 |
| 6 | -1 | 0.2679 | -1 |
| 7 | -1 | -0.2679 | -1 |
| 8 | -1 | -1 | -1.366 |
| 9 | -0.2679 | -1 | -1 |
| 10 | 0.2679 | -1 | -1 |
| 11 | 1 | -1 | -1.366 |
| 12 | 1 | -0.2679 | -1 |

# B. *The vertical problem via traditional CR Method*

For any , the distribution system subproblem can be expressed as:

|  |  |
| --- | --- |
|  | (1) |

where and .

According to [1], the following theorem holds:

Theorem 1: For the multi-parameter programming problem (1), we have:

1. The entire parameter space is convex;
2. The optimal solution is continuous and piecewise affine in the entire parameter space ;
3. The optimal value of the objective function is continuous and piecewise affine in the entire parameter space .

The Lagrangian function of (1) is given as:

|  |  |
| --- | --- |
| , | (2) |

its KKT conditions is given by

|  |  |
| --- | --- |
|  | (3) |

where indicates the active constraints in inequality constraints, while indicates the inactive constraints in inequality constraints.

After obtaining (3) and taking its inverse, each DSO can solve the local boundary region and provide the piecewise affine cost functions of the boundary variables of distribution systems in parallel and pass them to the TSO:

|  |  |
| --- | --- |
|  | (4) |

where the subscript represents the information exchanged between transmission system *i* and its connected distribution system *j*; the superscript represents the result of the *k*-th iteration. The detailed solution process of , , and can be found in [2].

After receiving the information passed by the DSO, the TSO solves the following problem:

|  |  |
| --- | --- |
|  | (5) |

where and .

图示

AI 生成的内容可能不正确。

Fig. 1. Traditional CR searching process.

Fig. 1 illustrates the traditional CR searching process. The TSO sends to each DSO, and the DSO returns the CR and the piecewise affine cost function to DSO, enabling the TSO to find the optimal solution in , which lies on the boundary of . To avoid the DSOs returning to redundant boundary regions, the TSO first moves a small step along the negative gradient direction, allowing DSOs to find new CRs with lower costs. The next parameter of boundary variable sent by the TSO is calculated as follows:

|  |  |
| --- | --- |
|  | (6) |

where  is the step size, is the projection matrix considering boundary constraints. The solution process can refer to [2].

The iteration stops when the following conditions are satisfied:

|  |  |
| --- | --- |
|  | (7) |

where is a small positive constant.

# C. *Detailed Settings of the Case studies*

Region A includes a modified IEEE 5-node TPS, a modified IEEE 15-node DPS and a modified IEEE 22-node DPS. In the TPS of region A, traditional generators are installed at nodes 1, 3, 4 and 5. The 15-node DPS features distributed PV generators at nodes 5, 10 and 15, while the 22-node DPS has distributed PV generators at nodes 5, 13, 17 and 20. Region B includes a modified IEEE 5-node TPS and two modified IEEE 15-node DPSs. In the TPS of region B, traditional generators are installed at nodes 6, 8 and 9, while its distribution systems lack any generators.

Specifically, the DD-BALADIN algorithm employs the ALADIN algorithm at the upper level and data-driven expanded CR at the lower level, while the Nested ADMM algorithm utilizes the ADMM algorithm at both levels. The Centralized AC-OPF method, on the other hand, solves the entire MRTDS system in a centralized manner. In terms of modeling, the DD-BALADIN algorithm applies to the linearized BCLA equation model for distribution systems and the AC-OPF model for transmission systems, whereas the Nested ADMM algorithm and the Centralized AC-OPF method adopts the AC-OPF model for both transmission and distribution systems.

References

1. F. Borrelli, Constrained Optimal Control of Linear and Hybrid Systems. New York, NY, USA: Springer, 2003, vol. 290.
2. Y. Guo, L. Tong, W. C. Wu, B. M. Zhang, and H. B. Sun, “Coordinated multi-area economic dispatch via critical region projection,” *IEEE Trans. Power Syst.*, vol. 32, no. 5, pp. 3736–3746, Sep. 2017.